

A simple criterion for effects beyond hard thermal loops

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Abstract

Thermal amplitudes with ultrasoft momenta, which are not accessible by standard methods of perturbation theory, have recently attracted a lot of interest. However, the comparison of external momenta with the ultrasoft scale $g^2 T \ln(1/g)$ is a too crude criterion, since amplitudes with hard external momenta can also be non perturbative, if these momenta are close enough to the light-cone. In this letter, I give a more refined criterion to decide if an amplitude is non perturbative, that applies to all situations. In physical terms, this condition states that non-perturbative effects appear if the particles running in loops have to travel distances larger than their mean free path.

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1 Introduction

Early attempts to calculate the damping rate of a gluon at rest by using the bare perturbative expansion of thermal QCD led to inconsistent results. It was realized by Braaten and Pisarski [1], and by Frenkel and Taylor [2] that the resummation of 1-loop corrections known as “hard thermal loops” (HTL in the following) is necessary because they are of the same order of magnitude as their tree-level counterparts for soft (momentum of order gT) external momenta. This leads to an effective perturbative

expansion [3] for soft modes, in which thermal effects appear via modifications of the propagators, and through non-local couplings. The physical origin of this non-locality is easy to understand: soft modes can couple to a hard (momentum of order T) thermalized parton of the plasma at different points along its trajectory.

In the HTL framework, the hard modes are massless and propagate freely in the plasma. Their masslessness has already been identified as a source of collinear singularities when one has to deal with amplitudes having external momenta close to the light-cone, as exemplified by the calculation of the production rate of real photons [4]. Flechsig and Rebhan [5] have shown that it is possible to slightly modify the definition of hard thermal loops by giving a thermal mass to the particle running in the loop, without altering the properties of HTLs.

HTLs can also be derived from kinetic theory [6], where they appear via a Vlasov (collisionless) equation. In this framework, it was found that collisions become important at the ultra-soft scale $g^2 T \ln(1/g)$ [7]. In more physical terms, such modes couple to very long wavelength density fluctuations of the hard modes, and the propagation of hard modes over long distances is affected by collisions.

However, there are examples of problems in which one has to take into account collisions even if the external momentum scale is hard. Among such problems is the production rate of hard photons, which is

sensitive to the collisions of hard quarks in the plasma [8], a difficulty very similar to the one encountered for ultra-soft amplitudes. This indicates that the condition $Q \lesssim g^2 T \ln(1/g)$ on the external momentum used in order to determine if collisions are important is too crude to apply to situations like the hard real photon production.

In this letter, I derive a more accurate criterion to decide if effects beyond the HTLs (like collisions) are important in a given problem. This condition reads $\lambda_{\text{mean}} \lesssim \lambda_{\text{coh}}(Q)$, where λ_{mean} is the mean free path of the particle running in the loop, and $\lambda_{\text{coh}}(Q)$ is a coherence length constructed with the external momentum Q . This condition reduces to $Q \lesssim g^2 T \ln(1/g)$ only in special cases.

2 Coherence length

Let me consider a generic diagram evaluated at finite temperature in which I isolate an arbitrary loop¹ having Q as one of its external momenta. Having in mind formalisms like the retarded-advanced formalism (or, equivalently, the imaginary time formalism), we know that this loop is evaluated by cutting each of the propagators of the loop in turn. Let me focus particularly on the term where the propagator carrying the momentum P (see Fig. 1) is cut.

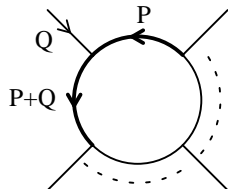


Figure 1: Generic configuration of momenta. The boldface line denotes the resummation of the width Γ on the propagator.

The piece of interest to us is the product of the propagator carrying the momentum P and the propagator carrying the momentum $R \equiv P + Q$ (i.e. the two propagators adjacent to the external line of mo-

mentum Q):

$$[\Delta_R(P) - \Delta_A(P)]\Delta_R(R). \quad (1)$$

The discontinuity in the square bracket is the cut propagator of momentum P . The dominant term is obtained when we keep the product of an advanced and a retarded propagator:

$$-\Delta_A(P)\Delta_R(R). \quad (2)$$

Indeed, it will turn out that these two propagators have very close poles separated by the real energy axis. As a consequence the integration contour cannot be moved to go around the poles, and this configuration leads to large contributions.

For the sake of generality, we assume the propagators in the loop to carry a mass M and a collisional width $\Gamma \sim g^2 T \ln(1/g)$. Since the loop momentum is usually hard, the mass M will typically be the asymptotic mass $M_\infty \sim gT$ introduced by Flechsig and Rebhan to improve the HTLs near the light-cone. The width Γ is a naive model for the collision term² of a Boltzmann equation, and sensitivity to this parameter in a loop indicates that we are in a regime where collisions are important. In this model, the inverse of the propagator reads $\Delta_{R,A}^{-1}(K) = (k_0 \pm i\Gamma)^2 - \mathbf{k}^2 - M^2$.

Picking the value of p_0 at the pole of $\Delta_A(P)$, and plugging it in the denominator of $\Delta_R(P+Q)$, we find:

$$\Delta_R^{-1}(R) \approx 2\omega q_0 - 2pq \cos \theta + Q^2 + 4i\Gamma(\omega + q_0), \quad (3)$$

where we have neglected terms in Γ^2 , and where θ is the angle between the vectors \mathbf{p} and \mathbf{q} . In the above equation, ω denotes the real part of the pole of $\Delta_A(P)$, i.e. $\omega \equiv \pm\sqrt{\mathbf{p}^2 + M^2}$.

At this stage, we already see that the parameter Γ can be important even for large Q , provided that $q \approx q_0$, and $\cos \theta \approx \pm 1$. In other words, the collisions manifest themselves in perturbation theory via

²Strictly speaking, the collision term of a Boltzmann equation does not necessarily lead to a complex pole in the propagator. It may happen that the propagator has a completely different analytical structure, without a pole [9]. Therefore, the width $\Gamma \sim g^2 T \ln(1/g)$ can be seen as a model of the effect of collisions. This model is supported by the fact that the spectral function of the exact propagator is very similar to the spectral function of the propagator modeled by a width [9].

¹This loop need not be a hard thermal loop.

collinear singularities, and Γ acts as a collinear regulator. This regulator can be extracted by setting $|\cos \theta| = 1$. Assuming also that $M \ll p, \omega$, but keeping the external momentum Q arbitrary, we obtain

$$\Delta_R^{-1}(R) \approx 2\omega q \left[1 - \text{sign}(\omega) \cos \theta + \frac{M_{\text{eff}}^2}{2\omega^2} \right], \quad (4)$$

where we denote³:

$$M_{\text{eff}}^2 \equiv M^2 + Q^2 \frac{\omega(2\omega + q + q_0)}{q_0(q + q_0)} + 4i\Gamma \frac{\omega(\omega + q_0)}{q_0}. \quad (6)$$

If $q_0 \approx q$, this effective mass is identical to the one introduced in [8].

We can see now that the width appears only in the imaginary part of this effective mass. As a consequence, the condition to have an effect due to collisions is simply $\text{Re } M_{\text{eff}}^2 \lesssim \text{Im } M_{\text{eff}}^2$, or

$$(2\Gamma)^{-1} \lesssim \left[\frac{q_0 \text{Re } M_{\text{eff}}^2}{2\omega(\omega + q_0)} \right]^{-1}. \quad (7)$$

The left hand side of this inequality is nothing but the mean free path λ_{mean} of the loop particle between two soft scatterings. It is also possible to give a simple physical interpretation to its right hand side. Indeed, the quantity in the bracket gives the difference $\Delta E \equiv r^0 - \omega_r$ between the energy and its on-shell value for the particle of momentum R . Its inverse is the typical lifetime of this virtual state, and is usually called coherence length and denoted by $\lambda_{\text{coh}}(Q)$. This quantity has the physical interpretation of the length traveled⁴ by the virtual particle of momentum $P + Q$ before it fragments into an on-shell particle of momentum P and the external particle of momentum Q . Therefore, the above inequality can be recasted in the more intuitive condition:

$$\lambda_{\text{mean}} \lesssim \lambda_{\text{coh}}(Q). \quad (8)$$

³If we were strictly in the HTL approximation, this quantity would be:

$$M_{\text{eff}}^2|_{\text{HTL}} = Q^2 \frac{2\omega^2}{q_0(q_0 + q)}, \quad (5)$$

since at this level of approximation we have $M = 0$, $\Gamma = 0$, and we neglect Q^2 compared to $2P \cdot Q$.

⁴By construction, the length scale obtained by setting $\cos \theta = \pm 1$ is a longitudinal scale.

Eq. (8) is the general condition for effects due to collisions and its physical interpretation is the following: there is a sensitivity to collisions if the loop particle travels distances larger than the typical distance between two successive collisions. In other words, $\lambda_{\text{coh}}(Q)$ gives a quantitative measure of “how much non-local” is the effective coupling associated to the loop.

One must also note that the above considerations must be repeated for each external leg. There is a sensitivity to collisions if any of the $\lambda_{\text{coh}}(Q_i)$ one can define with the external momenta Q_i is larger than the mean free path.

As a preliminary check, we see that in the particular limit of almost static fields ($q_0 \ll q$), this condition becomes $q \lesssim 2\Gamma \sim g^2 T \ln(1/g)$. In the opposite limit ($q \ll q_0$) where the external particle is at rest, the condition similarly becomes $q_0 \lesssim 2\Gamma \sim g^2 T \ln(1/g)$. Therefore, in these two limiting cases, Eq. (8) is equivalent to the usual condition $Q \lesssim g^2 T \ln(1/g)$ used in [7].

However, the momentum Q need not be ultra-soft in order for Eq. (8) to be satisfied. Indeed, it is trivial to see that if $Q^2 = 0$ we have $\lambda_{\text{coh}}^{-1}(Q) \approx M^2/2\omega$ if $q_0 \rightarrow +\infty$. Therefore, if $M \sim gT$, the condition is satisfied even for arbitrarily hard external momenta, provided they are on the light-cone. In order to make this discussion more visual, Fig. 2 represents contour curves of the quantity $\lambda_{\text{coh}}(Q)$ in the (q, q_0) plane. From this plot, one can readily see that hard mo-

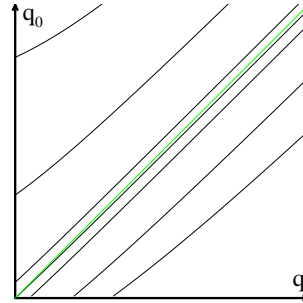


Figure 2: Contour curves of $\lambda_{\text{coh}}(Q)$. Large values of $\lambda_{\text{coh}}(Q)$ are packed in the vicinity of the light-cone.

menta close to the light-cone are as “dangerous” as

ultra-soft momenta.

In fact, the problem with hard momenta close to the light-cone could also have been guessed from kinetic theory, where one must compare the drift term $(v \cdot \partial_x) \delta N(k, x)$ and the collision term, which in the relaxation time approximation can be written as $-\Gamma \delta N(k, x)$. In momentum space, this comparison amounts to compare $v \cdot Q$ with Γ , and one sees again the relative importance of the collision term for large Q if $Q^2 \approx 0$.

3 Vertex corrections

Important corrections due to a collisional width on the loop propagators are not the only manifestation of collisions in perturbation theory. Some vertex corrections inside the loop also become important when the condition of Eq. (8) is satisfied. More specifically, these vertex corrections are ladder corrections connecting the two propagators adjacent to the external line carrying the momentum Q . Indeed, one can

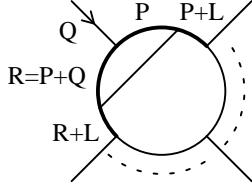


Figure 3: Dominant corrections to the loop due to collisions. A boldface propagator is a propagator where a width has been resummed.

easily see that each ladder correction increases the degree of collinear divergence in the diagram, which compensates the extra coupling constants. Each new rung in the ladder modifies the result by a factor that can be estimated to be (see Fig. 3 for the notations):

$$I \equiv g^2 pr \int \frac{d^4 L}{(2\pi)^4} \Delta_A(P+L) \Delta_R(R+L) n_B(l_0) \rho(L), \quad (9)$$

where L is the momentum flowing in the rung, $\rho(L)$ is the spectral function of the exchanged boson and $n_B(l_0)$ its statistical weight. Having in mind applications in QCD, we added the factor pr that gives

the correct order of magnitude for the numerator of quark propagators, or for the momentum dependence coming from the 3-gluon coupling. In order to estimate the integral, it is convenient to use the integration variables l_0 , the transverse (with respect to Q) momentum \mathbf{l}_\perp and the longitudinal momentum l_z . Performing the l_z integral in the complex plane by using the poles of $\Delta_A(P+L)$, we can estimate⁵

$$I \propto g^2 T \frac{pr}{q_0} \int d^2 \mathbf{l}_\perp \frac{dl_0}{l_0} \rho(l_0, \mathbf{l}_\perp) \frac{1}{(\mathbf{p}_\perp + \mathbf{l}_\perp)^2 + M_{\text{eff}}^2}, \quad (10)$$

where M_{eff}^2 is the effective mass introduced earlier. The value of l_z at the pole of $\Delta_A(P+L)$ being very small, we neglect the l_z dependence of the spectral function ρ . Because the second denominator contains $\mathbf{p}_\perp + \mathbf{l}_\perp$, the second loop cannot be factorized from the first one. However, the order of magnitude of \mathbf{p}_\perp is also M_{eff} (because of a denominator $\mathbf{p}_\perp^2 + M_{\text{eff}}^2$ in the first loop), so that for the sole purpose of estimating I we can just replace $\mathbf{p}_\perp + \mathbf{l}_\perp$ by \mathbf{l}_\perp . In addition, one can use sum rules to perform the l_0 integral:

$$I \sim g^2 T \frac{pr}{q_0} \int d^2 \mathbf{l}_\perp \frac{1}{\mathbf{l}_\perp^2 + \mu^2} \frac{1}{\mathbf{l}_\perp^2 + M_{\text{eff}}^2}, \quad (11)$$

where μ is a Debye mass (of order gT) if the exchanged gauge boson is longitudinal, or the magnetic mass (of order $g^2 T$) if this boson is transverse. Up to some inessential factors, the above integral is

$$I \sim g^2 T \ln \left(\frac{M_{\text{eff}}^2}{\mu^2} \right) \frac{pr}{q_0 (M_{\text{eff}}^2 - \mu^2)}. \quad (12)$$

Note that we have always $\mu \lesssim M_{\text{eff}}$. If we have $\mu \ll M_{\text{eff}}$ (this is what happens for a transverse gluon since then $\mu \sim g^2 T$), the above result simplifies into

$$I \sim g^2 T \ln \left(\frac{M_{\text{eff}}}{\mu} \right) \frac{pr}{q_0 M_{\text{eff}}^2} \sim \Gamma \frac{pr}{q_0 M_{\text{eff}}^2}. \quad (13)$$

⁵Here, we do not consider the poles of the spectral function $\rho(L)$. This approximation, known as the “approximation of independent scatterings”, is valid if the screening length μ^{-1} is smaller than the mean free path Γ^{-1} . This is not satisfied for transverse gluons, and the corresponding contribution has been studied in [10]. It is important when the coherence length becomes larger than the screening length, and is not related to the collinear singularities we are studying here.

If on the contrary we have $\mu \approx M_{\text{eff}}$ (which can happen for a longitudinal gluon), it becomes⁶

$$I \sim g^2 T \frac{pr}{q_0 M_{\text{eff}}^2} \sim \Gamma \frac{pr}{q_0 M_{\text{eff}}^2} . \quad (14)$$

We have now to consider two cases, depending on whether M_{eff}^2 is dominated by its real part or imaginary part:

$$\begin{aligned} \text{If } \lambda_{\text{coh}}(Q) \ll \lambda_{\text{mean}} , \quad I &\sim \frac{\lambda_{\text{coh}}(Q)}{\lambda_{\text{mean}}} \ll 1 , \\ \text{If } \lambda_{\text{mean}} \lesssim \lambda_{\text{coh}}(Q) , \quad I &\sim \frac{\lambda_{\text{mean}}}{\lambda_{\text{mean}}} = 1 . \end{aligned} \quad (15)$$

Therefore, we see from this estimate that each new rung in the ladder brings an extra contribution which is of order 1 if Eq. (8) is satisfied (both for transverse and longitudinal gluons, contrary to the term studied in [10]), indicating that ladder corrections must be resummed whenever the effect of the width Γ is important. It is worth noting that the mechanism that makes ladder corrections important in this context is related to collinear singularities (the variable l_{\perp} can be related to an angular deviation), that do not distinguish transverse and longitudinal gluons.

It may happen that the vertex corrections actually cancel the resummation of Γ [11]. This is not the case in QCD, or for some specific problems like the photon production rate by a quark-gluon plasma [8]. When they occur, these cancellations are the sign that the relevant mean free path is actually larger than the inverse Γ^{-1} of the damping rate (usually $1/g^4 T \ln(1/g)$ instead of $1/g^2 T \ln(1/g)$)⁷. The simple discussion presented in this paper cannot exclude such cancellations, and they must be studied on a case by case basis. In the language of kinetic theory, these cancellations appear immediately in the collision term of the Boltzmann equation.

⁶We can still identify Γ in the result, because if μ is large enough to have $\mu \approx M_{\text{eff}}$, then the damping rate Γ has an infrared cutoff large enough to suppress the logarithm.

⁷From this perspective, there seems to be a difference between photon production and momentum transport for instance: the relevant scatterings for momentum transport are hard and their rate is small ($g^4 T \ln(1/g)$), while photons (soft, or hard but collinear) can be produced in a quark gluon plasma via bremsstrahlung induced by soft scatterings of the quarks, which are more frequent (rate $g^2 T \ln(1/g)$).

As a side note, these vertex corrections were also to be expected on the basis of Ward identities in gauge theories. The fact that we obtain for them the same criterion as for the corrections by the width can therefore be seen as a consistency check.

4 Examples

In this section, I list a few examples where use of Eq. (8) can tell immediately if the problem is tractable by perturbative methods (i.e. without going beyond hard thermal loops) or not.

4.1 Hard Thermal Loops

The criterion $\lambda_{\text{mean}} \lesssim \lambda_{\text{coh}}(Q)$ can be applied to determine in which kinematical regime the hard thermal loops themselves should be corrected by effects due to collisions. We see that the situation $Q \lesssim g^2 T \ln(1/g)$ studied in [7] is not the only domain where such corrections are important. Indeed, HTLs should also be corrected for soft momenta⁸ close to the light cone. From a technical perspective, this is due to collinear singularities that show up in HTLs when they have external legs close to the light-cone, and the asymptotic mass M_{∞} advocated in [5] to solve this problem is not the most relevant regulator.

4.2 Viscosity

The shear viscosity has been calculated in a scalar theory by Jeon in [12] and more recently in [13], and it was noticed that this quantity receives contributions from an infinite series of diagrams. In thermal field theory, this transport coefficient is obtained as the imaginary part of the correlator of two energy-momentum tensors, in the limit $q = 0, q_0 \rightarrow 0$. Therefore, this corresponds to a point in figure 2 where the coherence length $\lambda_{\text{coh}}(Q)$ is infinite, which explains why this quantity is severely non-perturbative.

⁸The derivation of HTLs in [1] assumed generic soft external momenta, far enough from the light-cone.

4.3 Photon production

In [8], the photon production rate of quasi-real photons by a quark-gluon plasma has been found to be sensitive to multiple scatterings undergone by the quark that emits the photon, even for a hard photon. The criterion for this effect is precisely $\lambda_{\text{mean}} \lesssim \lambda_{\text{coh}}(Q)$ where Q is the momentum of the produced photon. In this particular situation, the effect of collisions is known as the Landau-Pomeranchuk-Migdal effect, and the coherence length $\lambda_{\text{coh}}(Q)$ has also the interpretation of the formation time of the photon.

4.4 Damping rate of a fast fermion

The perturbative calculation of the damping rate of a hard fermion suffers also from collinear singularities [14], which can be understood in the present framework because the condition Eq.(8) is satisfied for this problem (the fermion is hard, but on-shell so that the corresponding $\lambda_{\text{coh}}(Q)$ is large.). The non-perturbative study of this problem has been performed by Blaizot and Iancu in [9], and led to a non-exponential decay of the propagator for large time differences.

4.5 Out-of-equilibrium systems

As a side note, we can also mention another area where the coherence length defined in this paper should prove useful: that of out of equilibrium systems. New questions arise when the system is out-of-equilibrium: does it make sense to define a local (in space-time) production rate? can one ignore effects of the relaxation towards equilibrium in effective couplings? To answer these questions, one should compare the coherence length with the relaxation time of the system, or with the typical length scale of spatial inhomogeneities. Indeed, if the coherence length $\lambda_{\text{coh}}(Q)$ associated with the external leg of some effective coupling is larger than the relaxation time of the system, then this coupling should receive corrections reflecting the fact that the system is out of equilibrium. In imaged terms, the effective coupling is so non-local that it becomes sensitive to the large scale

of gradients in the system.

5 Conclusions

In thermal field theory, effects from collisions manifest themselves through collinear and/or infrared singularities that enhance otherwise suppressed higher order diagrams. By studying the behavior of some of those diagrams, we have derived a condition under which collisions provide important corrections to an amplitude having an external momentum Q , that generalizes the usual $Q \lesssim g^2 T \ln(1/g)$ used in previous works. This condition reads

$$\lambda_{\text{mean}} \lesssim \lambda_{\text{coh}}(Q) , \quad (16)$$

where λ_{mean} is the mean free path of the particle running in the loop, and $\lambda_{\text{coh}}(Q)$ is the typical length this particle has to propagate. The interpretation of the criterion is therefore straightforward: collisions are essential if the particle running in the loop travels distances larger than the average distance between two collisions.

This criterion applies to a wide range of problems that were known to be non-perturbative, including problems involving hard momenta (that would have been misleadingly classified as perturbative if one uses the criterion $Q \lesssim g^2 T \ln(1/g)$), highlighting common features of seemingly unrelated problems.

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